Mutual Inductance of Collinear Solenoids

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July 29, 2016

Abstract

Efficient and accurate approximations to solenoidal inductance exist. By breaking a given solenoid into two pieces, it is possible to compute the mutual coupling between two adjacent solenoids. Breaking a solenoid into three segments, it is possible to bootstrap the process to calculate the coupling between all three segments. From this it possible to create an accurate model for either an arbitrarily tapped inductor, or two collinear solenoids separated by an arbitrary gap.

1 Inductance of a single solenoid

Numerous equations to compute the inductance of a solenoid exist. An excellent overview of the subject is found in [1]

For most practical purposes, David Knight's formula is adequate. It has asymptotically exact behavior for both short and long solenoids with a maximum error of 265ppm. Given N turns, a diameter (d) and length (l) in centimeters, the inductance in microhenries is approximated by

$$L(d,l,N) = .002\pi d N^2 \left[\ln(1+\frac{\pi d}{2l}) + \frac{1}{2.3004 + 3.2219 l/d + 1.7793 (l/d)^2} \right]$$

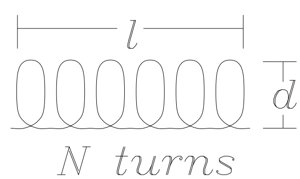

Figure 1.1: An inductor with length l, diameter d, N turns of wire.

2 Mutual inductance of two adjacent collinear solenoids

Given two adjacent solenoids of identical diameter d, lengths l_1, l_2, and turns of identical pitch, $N_2 = N_1 * l_2 / l_1$, we can calculate the inductance of each solenoid $L_1 = L(d, l_1, N_1)$, $L_2 = L(d, l_2, N_2)$,

[1] David W. Knight, An Introduction to the art of Solenoid Inductance Calculation with emphasis on radio-frequency application, Version 0.20 (unfinished), February 4th, 2016, available at http://g3ynh.info/zdocs/magnetics/.

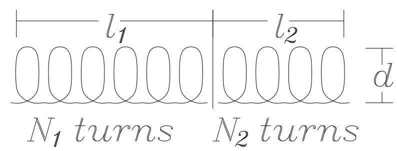

Figure 2.1: Two coupled collinear inductors.

plus the inductance of both solenoids in series $L_{12} = L(d, l_1 + l_2, N_1 + N_2)$. The equation for series coupled inductors with coupling coefficent k_{12} is

$$L_{12} = L_1 + L_2 + 2k_{12}\sqrt{L_1 L_2}$$

which can be solved to give

$$k_{12} = \frac{L_{12} - L_1 - L_2}{2\sqrt{L_1 L_2}}$$

3 Mutual inductance of two non-adjacent collinear solenoid

Given three adjacent solenoids of lengths l_1, l_2, l_3 it is possible to compute the inductance of each segment L_1, L_2, L_3, the inductance of each adjacent pair of segments in series L_{12}, L_{23}, and the inductance of the entire set of three inductors in series L_{123}.

There are three coupling constants k_{12}, k_{13}, k_{23} such that

$$L_{123} = L_1 + L_2 + L_3 + 2k_{12}\sqrt{L_1 L_2} + 2k_{23}\sqrt{L_2 L_3} + 2k_{13}\sqrt{L_1 L_3}$$

The coupling terms $k_{12}, k_{23}$are computed using the method of section 2 above. This leaves k_{13}as the only unknown to be solved for.

$$k_{13} = \frac{L_{123} - L_{12} - L_{23}}{2\sqrt{L_1 L_3}}$$

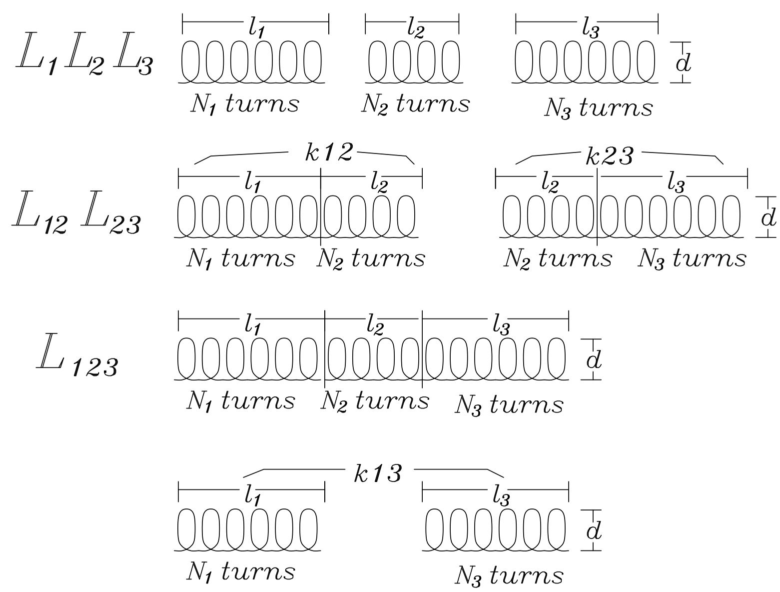

Figure 3.1: First line shows the definition of three different inductors. Second line shows inductors L1,2 combined to compute k12, and inductors L2,3 combined to compute k23. Third line shows all inductors combined into one large inductor L123. Last line computes the coupling k13 between two inductors L1,3 separated by the gap created by deleting L2.